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# DISCONTINUOUS SOLUTIONS OF GAS-DYNAMICS EQUATIONS TAKING INTO ACCOUNT THE RELAXATION OF A HEAT FLOW WITH A HEAT TRANSFER

**P. P. Volosevich,**[a] **I. I. Galiguzova,**[a] **E. I. Levanov,**[a] **and E. V. Severina**[b]

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*The gas-dynamics equations in Lagrangian mass coordinates for a heat flow with a relaxation and a hyperbolic heat transfer have been considered in the plane-symmetry approximation. The characteristics of the system of these equations were determined. Relations for the front of a strong discontinuity of its solution were obtained. With the theory of generalized solutions of quasi-linear equations, the stability of the discontinuities of gas-dynamic and heat quantities characteristic of the indicated flow was demonstrated.*

***Keywords:*** *gas dynamics of a heat flow with a heat transfer, stability of discontinuities of gas-dynamic and heat quantities, relaxation of a heat flow, relations for the front of a strong discontinuity.*

**Introduction.** The present work is devoted to investigating the stability of the discontinuous solutions of gas-dynamics equations for a heat flow subjected to relaxation. The system of equations being investigated is written in Lagrangian mass variables and is considered in the plane symmetry approximation. The characteristics of this system are determined and relations for the front of a discontinuity of its solution are derived. In [1, 2], the stability of strong discontinuities of the temperature and heat-transfer functions for an immovable medium was analyzed using the theory of generalized solutions of quasi-linear equations [3]. By analogy with this investigation it is shown below that the discontinuous solutions of the gas-dynamics equations for a heat flow with a relaxation and a heat transfer are stable. As the estimation criterion, we used the stability of the discontinuity of the solution of the system of $n$ linear equations in the theory of generalized solutions of quasi-linear equations.

**System of Gas-Dynamics Equations for a Heat Flow Subjected to Relaxation.** In the plane-symmetry approximation, gas-dynamics equations in mass Lagrangian coordinates have the form

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{\partial v}{\partial m}, \tag{1}$$

$$\frac{\partial v}{\partial t}=-\frac{\partial P}{\partial m}, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\varepsilon+\frac{v^2}{2}\right)=-\frac{\partial}{\partial m}(Pv+W)\,. \tag{3}$$

The expression for the heat flow $W$ is constructed with account for its relaxation (see [4–10] and the literature cited therein):

$$W=-K\frac{\partial T}{\partial m}-\tau\frac{\partial W}{\partial t}\,. \tag{4}$$

Equations (1)–(4) represent a hyperbolic system. Let us find characteristics of this system. First we will write some definitions for systems of quasi-linear equations of the first order

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[a]Institute of Mathematical Simulation, Russian Academy of Sciences, 4a Miusskaya Sq., Moscow, 125047, Russia; [b]Moscow Physical and Technical Institute, 9 Institute Lane, Dolgoprudnyi, Moscow Obl., 141700, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 82, No. 2, pp. 350–357, March–April, 2009. Original article submitted June 19, 2008.

$$\frac{\partial \mathbf{u}}{\partial t} + A\frac{\partial \mathbf{u}}{\partial m} = \mathbf{b}\,. \tag{5}$$

Let $\lambda = \lambda(m, t, \mathbf{u})$. It follows from the theory of quasi-linear equations that, to find all the eigenvalues of $\lambda$, it is necessary to solve the $n$-degree equation

$$\det(A - \lambda E) = 0\,. \tag{6}$$

The system of equations (5) is hyperbolic in the space $m$, $t$, $\mathbf{u}$ if the eigenvalues of $\lambda_1$, ..., $\lambda_n$ of the matrix $A$ are real and different everywhere in this region.

Let the equations of the ideal-gas state

$$P = \rho RT\,, \quad \varepsilon = \frac{RT}{\gamma - 1}\,, \quad \gamma > 1 \tag{7}$$

be true.

Then system (1)–(4) can be rearranged to a form analogous to (5):

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) - \frac{\partial v}{\partial m} = 0\,, \tag{8}$$

$$\frac{\partial v}{\partial t} - R\rho^2 T\frac{\partial}{\partial m}\left(\frac{1}{\rho}\right) + R\rho\frac{\partial T}{\partial m} = 0\,, \tag{9}$$

$$\frac{\partial T}{\partial t} + (\gamma - 1)\,\rho T\frac{\partial v}{\partial m} + \frac{\gamma - 1}{R}\frac{\partial W}{\partial m} = 0\,, \tag{10}$$

$$\frac{\partial W}{\partial t} + \frac{K}{\tau}\frac{\partial T}{\partial m} = -\frac{W}{\tau}\,. \tag{11}$$

In accordance with (5), we obtain that $\mathbf{u} = \{1/\rho, v, T, W\}$, $A = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -\rho^2 RT & 0 & \rho R & 0 \\ 0 & (\gamma-1)\rho T & 0 & (\gamma-1)R \\ 0 & 0 & K/\tau & 0 \end{pmatrix}$, and $\mathbf{b} = \{0, 0, 0, W/\tau\}$. Equation (6) can be written in the form

$$\det(A - \lambda E) = \begin{pmatrix} -\lambda & -1 & 0 & 0 \\ -\rho^2 RT & -\lambda & \rho R & 0 \\ 0 & (\gamma-1)\,\rho T & -\lambda & (\gamma-1)\,R \\ 0 & 0 & K/\tau & -\lambda \end{pmatrix} = 0\,, \tag{12}$$

i.e.,

$$\lambda^4 - \left(\frac{\gamma-1}{R}\frac{K}{\tau} + \gamma\rho^2 RT\right)\lambda^2 + \frac{(\gamma-1)\,K\rho^2 T}{\tau} = 0\,.$$

Using the expressions for the adiabatic and isothermal mass velocities of sound, $C_{\mathrm{i}} = \rho\sqrt{RT}$ and $C_\gamma = \rho\sqrt{\gamma\rho T} = C_{\mathrm{i}}\sqrt{\gamma}$, and for the velocity of propagation of heat disturbances, $C_T = \sqrt{\frac{(\gamma-1)K}{R\tau}}$ at $\tau \neq 0$, we present Eq. (12) in the form

$$\lambda^4 - \left(C_T^2 + C_\gamma^2\right)\lambda^2 + C_T^2 C_{\mathrm{i}}^2 = 0\,. \tag{13}$$

Solution of (13) gives the eigenvalues of the matrix $A$

$$\lambda_1 = \sqrt{0.5\,[C_T^2 + C_\gamma^2 + \sqrt{(C_T^2 + C_\gamma^2)^2 - 4C_T^2 C_i^2}\,]}\ ,\quad \lambda_2 = -\lambda_1\ ,$$

$$\lambda_3 = \sqrt{0.5\,[C_T^2 + C_\gamma^2 - \sqrt{(C_T^2 + C_\gamma^2)^2 - 4C_T^2 C_i^2}\,]}\ ,\quad \lambda_4 = -\lambda_3\ . \tag{14}$$

The equalities

$$\frac{dm}{dt} = \lambda_1\ ,\quad \frac{dm}{dt} = \lambda_2\ ,\quad \frac{dm}{dt} = \lambda_3\ ,\quad \frac{dm}{dt} = \lambda_4 \tag{15}$$

determine the four families of characteristics of the gas-dynamics equations (1)–(4) for a heat flow subjected to relaxation.

**Relations for the Front of a Strong Discontinuity.** Since system (1)–(4) is hyperbolic, the desired quantities admit a strong discontinuity. In this case, not only the gas-dynamic functions, but also the temperature $T = T(m,\ t)$ and the heat flow $W = W(m,\ t)$, can be discontinuous.

Let us introduce the auxiliary function $V = V(m,\ t)$ satisfying, at $\tau \neq 0$, the equation [6, 7]

$$\frac{\partial V}{\partial m} = \frac{K}{\tau}\frac{\partial T}{\partial m}\ . \tag{16}$$

Using (16), we rewrite Eq. (4) in the form

$$\frac{\partial W}{\partial t} = -\frac{\partial V}{\partial m} - \frac{W}{\tau}\ . \tag{17}$$

The relations representing the conservation laws at the front of a strong discontinuity can be obtained, by analogy with [11] (see also [12, 13], by formal integration of each of the equations of system (1)–(3), (17) over the small region (contracting to the zero volume) of change in the independent variables $m$ and $t$, including a discontinuity line. Let $m = m_{\text{lin}} = m_{\text{lin}}(t)$ and $D_{\text{m}} = dm_{\text{lin}}/dt$. Upon the above-indicated integration, we obtain the following relations, in which the quantities downstream of the discontinuity front are denoted by index 2 and the quantities upstream of it are denoted by index 1:

$$D_{\text{m}}\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right) = v_1 - v_2\ , \tag{18}$$

$$D_{\text{m}}\,(v_2 - v_1) = P_2 - P_1\ , \tag{19}$$

$$D_{\text{m}}\left(\varepsilon_2 + 0.5v_2^2 - \varepsilon_1 - 0.5v_1^2\right) = P_2 v_2 - P_1 v_1 + W_2 - W_1\ , \tag{20}$$

$$D_{\text{m}}\,(W_2 - W_1) = V_2 - V_1\ . \tag{21}$$

The function $V = V(m,\ t)$ can be determined from Eq. (16) in the process of solving a corresponding concrete problem.

Let the following conditions be fulfilled upstream of the discontinuity front:

$$v = v_1 = 0\ ,\quad T = T_1 > 0\ ,\quad V = V_1\ ,\quad W = W_1 = 0\ ,\quad P = P_1 = \rho_0 R T_1\ ,\quad \rho = \rho_1 = \rho_0\ . \tag{22}$$

For an immovable medium, from (20) and (21), with the use of the equations of state (7) we obtain

$$W_2 = \frac{D_m R}{\gamma - 1}(T_2 - T_1)\,, \tag{23}$$

$$V_2 - V_1 = W_2 D_m = \frac{D_m^2 R}{\gamma - 1}(T_2 - T_1)\,. \tag{24}$$

At $K = K(T)$ and $\tau = \tau(T)$, Eq. (16) takes the form

$$\frac{\partial V}{\partial T} = \frac{K(T)}{\tau(T)}\,. \tag{25}$$

Let

$$K = K_0 T^{a_0}\,,\quad \tau = \tau_0 T^{a_1}\,,\quad a_0 > a_1 > 0\,. \tag{26}$$

Upon integrating (25) with respect to the quantities determined by (26) on the assumption that $V = V_1$ at $T = T_1$ and $a_0 - a_1 + 1 \neq 0$, we obtain

$$V = V_1 + \frac{K_0}{\tau_0 (a_0 - a_1 + 1)}\left(T^{a_0 - a_1 + 1} - T^{a_0 - a_1 + 1}\right)\,. \tag{27}$$

Let us assume that $a_0 - a_1 = 1$. Then the relation determining the temperature $T = T_2$ at the discontinuity front will take the form

$$T_2 = \frac{2\tau_0 R D_m^2}{K_0 (\gamma - 1)} - T_1\,. \tag{28}$$

In a moving medium ($v_0 \neq 0$), assuming that $\rho = \rho_0/\eta$, from Eqs. (18)–(22) we obtain

$$\eta_2 = \frac{\rho_0}{\rho_2}\,,\quad \eta_1 = 1\,, \tag{29}$$

$$v_2 = \frac{D_m}{\rho_0}(1 - \eta_2)\,, \tag{30}$$

$$P_2 = \frac{D_m^2}{\rho_0}(1 - \eta_2) + P_1\,, \tag{31}$$

$$T_2 = \eta_2 \left(T_1 + \frac{D_m^2}{\rho_0}(1 - \eta_2)\right), \tag{32}$$

$$W_2 = \frac{1}{2}\,\frac{\gamma + 1}{\gamma - 1}\,\frac{D_m^3}{\rho_0^2}(1 - \eta_2)\left(\eta_2 - \frac{\gamma - 1}{\gamma + 1} - \frac{2\gamma}{\gamma + 1} + \frac{R\rho_0^2 T_1}{D_m^2}\right), \tag{33}$$

$$V_2 = V_1 + W_2 D_m\,. \tag{34}$$

**Stability of Discontinuous Solutions.** The stability of strong discontinuities of the temperature and heat-flow functions satisfying Eqs. (3) and (4) for an immovable medium was investigated in [1, 5], where this stability was analyzed using the theory of generalized solutions of quasi-linear equations [3]. An analogous method can be used for determining the discontinuous solutions of the system of gas-dynamics equations (1)–(4) at $\tau_0 > 0$.

For definiteness, we will assume that the desired functions describe a traveling wave with a strong discontinuity at its front.

Let the gas-dynamic and heat quantities satisfying the system of equations (1)–(3), (16), (17) have the form

$$F = F(m, t) = F(Dt - m)\,, \tag{35}$$

where $D$ = const. It will be assumed that the heat-conductivity coefficient $K$ and the relaxation time of the heat flow $\tau$ are power functions of the temperature $T$ and density $\rho$:

$$K = K_0 T^{a_0} \rho^{b_0}\,, \quad \tau = \tau_0 T^{a_1} \rho^{b_1}\,, \quad a_0 > a_1 > 0\,. \tag{36}$$

The corresponding independent variables and desired functions will be represented in the dimensionless form

$$x = \frac{Dt - m}{K_0 D^{2a_0 - 1} R^{-(a_0+1)} \rho_0^{-1a_0 + b_0}}\,, \quad \eta = \eta(x) = \frac{\rho_0}{\rho(m, t)}\,, \quad \alpha = \alpha(x) = \frac{v(m, t)}{D\rho_0^{-1}}\,, \quad \beta = \beta(x) = \frac{P(m, t)}{D^2 \rho_0^{-1}}\,,$$
$$f = f(x) = \frac{t(m, t)}{R^{-1} D^2 \rho_0^{-2}}\,, \quad \omega = \omega(x) = \frac{W(m, t)}{D^3 \rho_0^{-2}}\,, \quad \tilde{V} = \tilde{V}(x) = \frac{V(m, t)}{D^4 \rho_0^{-2}}\,. \tag{37}$$

The position of the front of the traveling wave will be determined by the quantity $m = Dt$ $(x = 0)$. In a perturbed medium, $m \le Dt$ $(x \ge 0)$.

It will be assumed that the traveling-wave front represents a strong discontinuity realized on conditions (22), (29)–(34), where $D_{\mathrm{m}} = D$. In variables (37), these conditions take the form

$$\alpha = \alpha_1 = 0\,, \quad f = f_1\,, \quad \tilde{V} = \tilde{V}_1\,, \quad \omega = \omega_1 = 0\,, \quad \beta = \beta_1 = f_1\,, \quad \eta = \eta_1 = 1\,; \tag{38}$$

$$\alpha_2 = 1 - \eta_2\,, \quad f_2 = \eta_2 (f_1 + 1 - \eta_2)\,, \quad \beta_2 = f_2 \eta_2^{-1}\,; \tag{39}$$

$$\omega_2 = \frac{1}{2}\frac{\gamma + 1}{\gamma - 1}(1 - \eta_2)\left(\eta_2 - \frac{1}{\gamma + 1}(\gamma - 1 + 2\gamma f_1)\right), \quad \tilde{V}_2 = \tilde{V}_1 - \omega_2\,. \tag{40}$$

The following formulas are obtained from Eqs. (1)–(3) in terms of variables (37) for the dimensionless functions of the velocity $\alpha = \alpha(x)$, the pressure $\beta = \beta(x)$, the temperature $f = f(x)$, and the heat flow $\omega = \omega(x)$ in the region $x \ge 0$:

$$\alpha = 1 - \eta\,, \quad \beta = f_1 + 1 - \eta\,, \quad f = \eta(f_1 + 1 - \eta)\,, \quad \omega = \frac{1}{2}\frac{\gamma + 1}{\gamma - 1}(1 - \eta)\left(\eta - \frac{1}{\gamma + 1}(\gamma - 1 + 2\gamma f_1)\right). \tag{41}$$

At $x = 0$, relations (41) satisfy both conditions (38) and the discontinuity conditions (39) and (40). Using (41) in variables (37), we represent Eq. (16) determining the function $V = V(m, t)$ in the form

$$\frac{d\tilde{V}}{d\eta} = \frac{1}{\varphi_0}\eta^{a_0 - a_1 - b_0 + b_1}(f_1 + 1 - \eta)^{a_0 - a_1}(f_1 + 1 - 2\eta)\,, \tag{42}$$

where $\varphi_0 = \dfrac{\tau_0 R^{a_0-a_1+1}}{K_0 D^{2(a_0-a_1-1)} \rho_0^{-2(a_0-a_1)+b_0-b_1}}$ is a dimensionless constant. With the use of (42), Eq. (17) can be written

$$\frac{d\eta}{dx} = \frac{1}{2}\frac{\gamma+1}{\gamma-1}\frac{(1-\eta)\left(\eta - \dfrac{\gamma-1+2\gamma f_1}{\gamma+1}\right)}{(f_1+1-\eta)^{a_0}(f_1+1-2\eta)\,\eta^{a_0-b_0} + \varphi_0\dfrac{\gamma+1}{\gamma-1}\left(\eta - \dfrac{\gamma(1+f_1)}{\gamma+1}\right)(f_1+1-\eta)^{a_1}\eta^{a_1-b_1}} . \tag{43}$$

An analysis has shown that, at $\varphi_0 = 0$ (the heat transfer is realized by the Fourier law $W = -K\frac{\partial T}{\partial m}$), the solution of Eq. (43) in the neighborhood of the "initial front" (38) is continuous [12, 14, 15]. In this case, condition (38) is fulfilled at a finite value of $x$ (at $x = 0$ in the case being considered) if the temperature upstream of the traveling-front wave is equal to zero: $f_1 = 0$. In the case where $f_1 \neq 0$, condition (38) is fulfilled only at $x = -\infty$. However, if at $\varphi_0 > 0$ the solution admits a strong discontinuity at the wave front, conditions (38) and (39) can be fulfilled at a finite value of $x$, in particular, at $x = 0$. In this case, the function $\eta = \eta(x)$, satisfying Eq. (43), varies in the region $x \geq 0$. At $x = 0$, the conditions $\eta = \eta_2$ and $0 < \eta_2 < 1$ should be fulfilled.

The squares of the velocities of sound can be written in terms of (37) as

$$C_{\mathrm{i}}^2 = D^2 f \eta^{-2} , \quad C_\gamma^2 = \gamma D^2 f \eta^{-2} , \quad C_T^2 = \frac{\gamma-1}{\varphi_0} D^2 f^{a_0-a_1} \eta^{-b_0+b_1} . \tag{44}$$

It will be assumed that

$$a_0 - a_1 = 1 , \quad b_0 = 1 , \quad b_1 = -1 . \tag{45}$$

In particular, conditions (45) are true for the coefficients $K$ and $\tau$ determined for a completely ionized plasma [16, 17]: $K = K_0 T^{5/2} \rho$ and $\tau = \tau_0 T^{3/2} \rho^{-1}$.

Using (44) and (45), we represent the eigenvalues of $\lambda_j$, $j = 1, 2, 3, 4$, in the form

$$\lambda_1 = A_0 D \eta^{-1}\sqrt{f} , \quad \lambda_2 = -A_0 D \eta^{-1}\sqrt{f} , \quad \lambda_3 = B_0 D \eta^{-1}\sqrt{f} , \quad \lambda_4 = -B_0 D \eta^{-1}\sqrt{f} , \tag{46}$$

where

$$A_0 = \sqrt{\frac{1}{2}\left(\frac{\gamma-1}{\varphi_0} + \gamma + \sqrt{\left(\frac{\gamma-1}{\varphi_0} + \gamma\right)^2 - 4\,\frac{\gamma-1}{\varphi_0}}\right)} ;$$

$$B_0 = \sqrt{\frac{1}{2}\left(\frac{\gamma-1}{\varphi_0} - \gamma + \sqrt{\left(\frac{\gamma-1}{\varphi_0} + \gamma\right)^2 - 4\,\frac{\gamma-1}{\varphi_0}}\right)} ; \quad \varphi_0 = \frac{\tau_0 R^2}{K_0} . \tag{47}$$

When the conditions (45) are fulfilled, Eq. (42) takes the form

$$\frac{d\tilde{V}}{d\eta} = \frac{1}{\varphi_0}\,\eta^{-1}\,(f_1+1-\eta)\,(f_1+1-2\eta) . \tag{48}$$

We will determine the function $\tilde{V} = \tilde{V}(x)$ from Eq. (48) on the assumption that, at $\eta = 1$ ($f = f_1$), the condition $\tilde{V} = \tilde{V}_1$ is fulfilled:

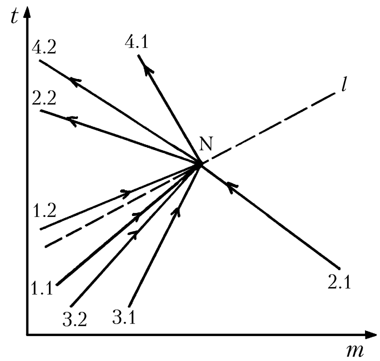


Fig. 1. Characteristics at the point N.

$$\tilde{V} = \tilde{V}_1 + \frac{1}{\varphi_0}\left((1-\eta)(3f_1 + 2 - \eta) + (f_1+1)^2 \ln \eta\right). \tag{49}$$

The law of propagation of the front of a traveling wave is as follows:

$$\frac{dm_{\text{lin}}}{dt} = D. \tag{50}$$

In the theory of solving the systems of $n$ quasi-linear equations, the conditions for the stability of a discontinuity are formulated in the following way [3] (see also [1, 5]). The $n+1$ characteristics should enter each point of the discontinuity trajectory $m = m_{\text{lin}}(t)$ and, correspondingly, the $n-1$ characteristics should depart from it.

Let N be a point on the discontinuity line. From (46) and (47), the characteristics corresponding to the values of the desired quantities downstream and upstream of the discontinuity front can be determined. At $\eta = \eta_2$ and $f = f_2 = \beta_2\eta_2$, we obtain that at the point N

$$\frac{dm_{1,2}}{dt} = A_0 D\eta_2^{-1}\sqrt{f_2} = A_0 D\sqrt{\beta_2\eta_2^{-1}}, \quad \frac{dm_{2,2}}{dt} = -A_0 D\sqrt{\beta_2\eta_2^{-1}},$$

$$\frac{dm_{3,2}}{dt} = B_0 D\sqrt{\beta_2\eta_2^{-1}}, \quad \frac{dm_{4,2}}{dt} = -B_0 D\sqrt{\beta_2\eta_2^{-1}} \tag{51}$$

and, at $f = f_1 = \beta_1$ and $\eta_1 = 1$,

$$\frac{dm_{1,1}}{dt} = A_0 D\sqrt{f_1} = A_0 D\sqrt{\beta_1}, \quad \frac{dm_{2,1}}{dt} = -A_0 D\sqrt{\beta_1}, \quad \frac{dm_{3,1}}{dt} = B_0 D\sqrt{\beta_1}, \quad \frac{dm_{4,1}}{dt} = -B_0 D\sqrt{\beta_1}. \tag{52}$$

The index $k$ in the expressions $dm_{k,j}/dt$ in formulas (51) and (52) corresponds to the eigenvalue of $\lambda_k$ ($k$ = 1, 2, 3, 4) and the index $j$ corresponds to the values of the desired quantities upstream of the discontinuity front ($j$ = 1) and downstream of it ($j$ = 2). In this case, $\beta_2 > \beta_1$, $0 < \eta_2 < 1$, and $A_0 > B_0$.

The quantities at the discontinuity front are determined by formulas (38)–(40). Using (49), from (40) we will obtain the transcendental equation

$$(1-\eta_2)(3f_1 + 2 - \eta_2) + (f_1+1)^2 \ln \eta_2 = \frac{1}{2}\varphi_0 \frac{\gamma+1}{\gamma-1}(1-\eta_2)\left(\eta_2 - \frac{\gamma - 1 + 2\gamma f_1}{\gamma+1}\right). \tag{53}$$

The characteristics intersecting the discontinuity line $l$ ($dm_{\text{lin}}/dt = D$) at the point N, presented in Fig. 1, were constructed for $\gamma = 5/3$, $a_0 - a_1 = 1$, $b_0 = 1$, $b_1 = -1$, $\eta_2 = 0.75$, and, respectively, $f_1 = \beta_1 = 0.1$, $\varphi_0 = \tau_0 R^2/K_0 \approx 0.21$. In this case, the line 1.1 — $dm_{1,1}/dt = \lambda_{1,1}$, the line 1.2 — $dm_{2,1}/dt = \lambda_{2,1}$, the line 1.3 — $dm_{3,1}/dt = \lambda_{3,1}$, the line 1.4 — $dm_{4,1}/dt = \lambda_{4,1}$, the line 2.1 — $dm_{1,2}/dt = \lambda_{1,2}$, the line 2.2 — $dm_{2,2}/dt = \lambda_{2,2}$, the line 2.3 — $dm_{3,2}/dt =$

$\lambda_{3,2}$, and the line 2.4 — $dm_{4,2}/dt = \lambda_{4,2}$. An analysis has shown that, at $n = 4$, $n + 1 = 5$ characteristics enter the point N and $n - 1 = 3$ characteristics depart from it. This means that the strong discontinuities of the desired functions being considered are stable.

**Conclusions.** The system of gas-dynamics equations (1)–(4) in Lagrangian mass variables for a heat flow with a relaxation has been considered in the plane-symmetry approximation. The characteristics of system (14) and the laws at the front of a strong discontinuity (29)–(34) were determined. It was shown on the basis of these results that the stability condition is fulfilled for this problem in the formulation of the theory of generalized solutions of quasi-linear equations (in our case, five characteristics enters a point on the discontinuity line and three characteristics depart from it when $n = 4$). Thus, it has been established that the strong discontinuity of the solutions of the system of gas-dynamics equations (1)–(4) is stable.

## NOTATION

A = $\{a_{ij}(m, t, \mathbf{u})\}$, $i = 1, 2, ..., n$, matrix of the $n$ order; $A_0$, $B_0$, $\varphi_0$, numerical coefficients determined by formula (47); $a_0$, $a_1$, exponents of the temperature dependence; $\mathbf{b} = \{b_1(m, t, \mathbf{u}), ..., b_n(m, t, \mathbf{u})\}$, vector of the right side; $b_0$, $b_1$, exponents of the density dependence; $C_i$, adiabatic velocity of sound; $C_\gamma$, isothermic velocity of sound; $C_T$, velocity of propagation of heat disturbances; $D$, velocity of a traveling wave; $D_m$, mass velocity of a discontinuity; $E$, unit matrix; $f$, dimensionless temperature dependence; $K$, heat-conductivity coefficient depending on the density and temperature; $l$, discontinuity line; $m$, mass Lagrangian coordinate; $m_{lin} = m_{lin}(t)$, wake of the discontinuity surface in the plane $(m, t)$; $P$, pressure in Lagrangian variables; $R$, universal gas constant; $t$, Lagrangian time coordinate; $T$, temperature in Lagrangian variables; $\mathbf{u}\{u_1(m, t), ..., u_n(m, t)\}$, vector-function of the independent variables $m$ and $t$; $v$, velocity of the gas in Lagrangian variables; $\tilde{V}$, dimensionless function determined by expression (37); $W$, heat flow in Lagrangian variables; $x$, dimensionless variable; $\alpha$, dimensionless function of velocity; $\beta$, dimensionless function of pressure; $\gamma$, constant relation between the specific heats; $\varepsilon$, specific internal energy in Lagrangian variables; $\eta$, dimensionless function of the specific volume; $\lambda$, eigenvalue of the matrix $A$; $\rho_0$, initial density upstream of the traveling-wave front; $\rho$, density in Lagrangian variables; $\tau$, relaxation time of the heat flow depending on the density and temperature; $\omega$, dimensionless function of the heat flow. Subscripts: m, mass; i, isoentropic; lin, line.

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